NONLOCALLY ELASTIC FILTRATION REGIME AND PRESSURE RECOVERY IN DEEP FORMATIONS

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We examined axisymmetric filtration flows in the nonlocally elastic filtration regime. The solution is presented of the problem of pressure recovery in a formation after sudden termination of fluid removal. It is shown that the well pressure recovery curve has two asymptotic rectilinear segments corresponding to different formation piezoconductivity coefficients (large for small times, and small for large times), which opens up the possibility of new interpretations of the known shapes of the measured curves.

1. In accordance with the theory of the nonlocally elastic filtration regime of homogeneous fluid in a deep planar formation [1], the linear equation of unsteady filtrational flow

$$(a_\rho + a)\frac{\partial p}{\partial t} - b\frac{\partial \sigma}{\partial t} = \frac{k_0}{m_0\mu_0}\nabla^2 p + \frac{G}{m_0\rho_0} \tag{1.1}$$

is supplemented by the integral condition for constancy of the rock pressure $\Gamma(x_1, x_2)$ at each point of the formation

$$\sigma(x_1, x_2; t) + \iint\limits_{-\infty}^{\infty} \Phi\left(\frac{x_i - \xi_i}{d}\right) p(\xi_i, t)\, d\xi_1\, d\xi_2 = \Gamma(x_1, x_2) \tag{1.2}$$

Here ρ and μ are the density and viscosity of the filtering fluid, k and m are the permeability and porosity of the formation, p is the pore pressure, and σ is the effective pressure in the skeleton of the porous medium

$$\rho/\rho_0 = 1 + a_\rho(p - p_0); \quad m/m_0 = 1 + a(p - p_0) - b(\sigma - \sigma_0), \quad k_0 = \text{const};$$

the subscript zero relates to the undisturbed state, G is the intensity of the sources (sinks) simulating well operation, and Φ is the influence function, given [1] in the form

$$\Phi\left(\frac{x_i - \xi_i}{d}\right) = \frac{1}{\pi d^2}\exp\left\{-\sum_{i=1,2}\frac{(x_i - \xi_i)^2}{d^2}\right\} \tag{1.3}$$

If the influence zone scale d is much smaller than the characteristic dimension of the region of variation of the pressure p, condition (1.2) becomes the usual [2, 3] local condition for constancy of the rock pressure: $\sigma + p = \Gamma$. However, if the scale d is relatively large, condition (1.2) reduces to the condition $\partial\sigma/\partial t = 0$ as $d \to \infty$.

The condition (1.2) is given the following physical intepretation [1]: for relatively small dimensions of the depression funnel the roof and floor of the formation do not distort and there is no compression of the skeleton of the medium ($\sigma = \sigma_0$). As the depression funnel grows the roof and floor begin to deflect, the load on the skeleton of the formation at the considered point x_1, x_2 increases and at the same time the effective compressibility of the formation increases.

2. If we write the condition (1.2), (1.3) in the (r, φ) polar coordinate system and then use the condition of independence of the local increments $\Delta\sigma(r, t)$, $\Delta p(r, t)$ on the polar angle φ, which is characteristic for flows with axial symmetry, and if we then use the integral (see [4], page 972)

$$\int\limits_0^{2\pi} \exp\left(\frac{2r\rho}{d}\cos\varphi\right) d\varphi = 2\pi I_0\left(\frac{2r\rho}{d^2}\right) \tag{2.1}$$

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we obtain

$$\Delta\sigma(r,t)+\frac{2}{d^2}\int_0^\infty I_0\left(2\frac{r\rho}{d^2}\right)\exp\left(-\frac{r^2+\rho^2}{d^2}\right)\Delta p(\rho,t)\rho d\rho=0 \tag{2.2}$$

where I_0 is the Bessel function of imaginary argument.

Let us examine the problem of pressure recovery in a formation after instantaneous shutdown of a well operating with the flowrate Q = const. Let $p_0(r)$ be the initial steady-state pressure distribution. The initial and boundary conditions are

$$p=p_0(r) \text{ for } t=0 \text{ and for } r\to\infty,\ \partial p/\partial r=0 \text{ for } r=0,\ t>0 \tag{2.3}$$

In this case the initial pressure distribution $p_0(r)$ satisfies the condition

$$\frac{2\pi kh}{\mu}\left(r\frac{\partial p_0}{\partial r}\right)=Q \quad \text{for} \quad r=0,\ t<0 \tag{2.4}$$

We set

$$p=p_0(r)+\frac{Q\mu}{2\pi kh}u(r,t) \tag{2.5}$$

Substituting (2.5) into (1.1), setting therein G = 0,and taking (2.2)-(2.4) into account, we obtain the solution of the integrodifferential equation

$$(1-\alpha)\frac{\partial u}{\partial t}-\frac{2\alpha}{d^2}\int_0^\infty I_0\left(2\frac{r\rho}{d^2}\right)\exp\left(-\frac{r^2+\rho^2}{d^2}\right)\frac{\partial u(\rho,t)}{\partial t}\rho\, d\rho=\frac{\varkappa}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{2.6}$$

with the conditions

$$u=0 \text{ for } t=0 \text{ and for } r\to\infty,\ (r\,\partial u/\partial r)=-1 \text{ for } r=0,\ t>0 \tag{2.7}$$

Here $\varkappa=k_0(\mu_0\beta m_0)^{-1}$ is the piezoconductivity coefficient, $\alpha=b/\beta$, $\beta=a_\rho+a+b$ is the maximal compressibility of the formation.

Applying to (1.6) the integral Hankel transformation relative to the transformant

$$U(\lambda,t)=\int_0^\infty u(r,t)J_0(\lambda r)r dr$$

we obtain the equation

$$\frac{\partial U}{\partial t}+\varphi(\lambda)U=\frac{\varphi(\lambda)}{\lambda^2},\quad \varphi(\lambda)=\frac{\varkappa\lambda^2}{1-\alpha[1-\exp(-{}^1\!/_4 d^2\lambda^2)]},\quad U=0 \text{ for } t=0 \tag{2.8}$$

The solution of (2.8) has the form

$$U(\lambda,t)=\frac{1-\exp[-\varphi(\lambda)t]}{\lambda^2},\qquad u(r,t)=\int_0^\infty\frac{1-\exp[-\varphi(\lambda)t]}{\lambda}J_0(r\lambda)d\lambda \tag{2.9}$$

and can be used to interpret well pressure recovery curves, i.e., the function $p(r_W, t)$ where r_W is the well radius. In this case we shall consider, as usual, that inside the real well there is a point (fictitious) well of the same flowrate, which can simulate the real well provided $r_W^2/(\varkappa t)\ll 1$ − relative smallness of the well radius − which is always realized in practice.

From the solution (2.9) we have

$$u(r_W t)=\int_0^\infty\frac{1-e^{-\psi(z,\tau;\theta,\alpha)}}{z}J_0(z)dz,\qquad \psi=\frac{\tau z^2}{1-\alpha[1-\exp(-\theta z^2)]} \tag{2.10}$$

where $\theta=d^2/(4r_W^2)$, $\tau=\varkappa t/r_W^2$. The quantity $\varepsilon=\alpha[1-\exp(-\theta z^2)]<1$, since $\alpha<1$, $\theta>0$. We note that

$$\psi=\frac{\tau z^2}{1-\varepsilon}=\tau z^2\sum_{n=0}^\infty\varepsilon^n,\qquad e^{-\psi}=e^{-\tau z^2}+e^{-\tau z^2}\sum_{n=1}^\infty\varepsilon^n\sum_m\frac{(-1)^m(\tau z^2)^m}{i!\,j!\ldots k!} \tag{2.11}$$

where the summation over m extends to all the positive integer solutions of the equations $i+2j+\ldots+lk=n$, $i+j+\ldots+k=m$ (see [4], page 34). Moreover the expansion holds

$$\varepsilon^n=\alpha^n(1-e^{-\theta z^2})^n=\alpha^n\sum_{\nu=0}^\infty\frac{(-1)^\nu n!}{\nu!(n-\nu)!}e^{-\nu\theta z^2} \tag{2.12}$$

Substituting the series (2.11), (2.12) into the integral (2.10) and integrating, we obtain

$$u=-\frac{1}{2}\,\mathrm{Ei}\left(-\frac{1}{4\tau}\right)-\sum_{n=1}^{\infty}\alpha^n\sum_m\frac{(-1)^m\tau^m}{i!\,j!\ldots k!}\sum_{\nu=0}^{n}\frac{(-1)^\nu n!\,(m-1)!}{\nu!\,(n-\nu)!}\times\frac{\exp(-{}^1/_8(\tau+\nu\theta))}{(\tau+\nu\theta)^{m-1/2}}M_{m-1/2}\left(\frac{1}{8(\tau+\nu\theta)}\right)\tag{2.13}$$

where we have introduced the Whittaker function [4]

$$M_{m-1/2}(x)=\frac{x^{1/2}e^{1/2x}}{(m-1)!}\frac{d^{(m-1)}}{dx^{(m-1)}}(x^{m-1}e^{-x}),\quad m\geqslant 1$$

In the limiting cases with $\theta=0$ and $\theta=\infty$ we have, respectively,

$$u_0=-\frac{1}{2}\,\mathrm{Ei}\left(-\frac{1}{4\tau}\right),\qquad u_\infty=-\frac{1}{2}\,\mathrm{Ei}\left(-\frac{1-\alpha}{4\tau}\right)\tag{2.14}$$

We shall make a rough estimate of the approach of the function $u(\tau)$ to these limiting values for arbitrary θ. According to [4] we have for sufficiently large $\tau+\nu\theta$.

$$M_{m-1/2}\left(\frac{1}{8(\tau+\nu\theta)}\right)=\frac{1}{2\sqrt{2}(\tau+\nu\theta)^{1/2}}+O\left(\frac{1}{(\tau+\nu\theta)^{3/2}}\right)$$
$$u=u_0-\frac{1}{2\sqrt{2}}\sum_{n=1}^{\infty}\alpha^n\sum_m\frac{(-1)^m(m-1)!}{i!\,j!\ldots k!}\sum_{\nu=0}^{n}\frac{(-1)^n n!}{\nu!\,(n-\nu)!}\left(1+\nu\frac{\theta}{\tau}\right)^{-m}+O\left(\frac{1}{\tau}\right)\tag{2.15}$$

If $\theta\ll\tau$, then

$$1+\nu\theta/\tau\approx 1$$

for the principal terms of the expansion (2.15). Then by virtue of the equality

$$\sum_{\nu=0}^{n}\frac{(-1)^\nu n!}{\nu!\,(n-\nu)!}=0$$

we obtain

$$u=u_0+O(1/\tau)\qquad\text{for}\quad\tau\gg 0\tag{2.16}$$

At the initial moments of time $\theta\gg\tau$. Then in (2.15) the principal terms will be those with $\nu=0$ and the terms with $\nu\neq 0$ are of order $O(\tau_n/\theta)$ and higher. Therefore, retaining in (2.14) only the first term $\varepsilon^n=\alpha^n$, we obtain $\psi=\tau z^2(1-\alpha)^{-1}$, which yields $u=u_\infty$. Hence we finally have the estimate

$$u=u_\infty+O(\tau/\theta)\quad\text{for}\ /\tau\ll\theta\tag{2.17}$$

3. According to (2.5) the pressure rise $\Delta p=p(r_w,t)-p(r_w,0)$ recorded in real wells is expressed in terms of the resulting solution for $u(\tau)$. In accordance with the above discussion we can identify three characteristic segments of the pressure recovery curve. The first segment 1 corresponds to the time interval

$$0\leqslant\tau<0.1\theta$$
$$\Delta p=-\frac{Q\mu}{4\pi kh}\,\mathrm{Ei}\left(-\frac{1-\alpha}{4\tau}\right)\approx\frac{Q\mu}{4\pi kh}\left(\ln\frac{2.25}{1-\alpha}+\ln\tau\right)\tag{3.1}$$

The second segment 2 corresponds to the interval $0.1\theta<\tau<10\theta$. Here we shall describe the pressure change approximately by three terms of the expansion (2.13), corresponding to terms of order ε^0, ε^1, ε^2, ε^3 in (2.11)

$$\begin{aligned}\Delta p\approx\frac{Q\mu}{2\pi kh}\Big(&\ln\frac{2.25}{1-\alpha}\tau-\frac{\alpha[1+2\alpha+3\alpha^2]}{2(1+\theta/\tau)}+\frac{\alpha^2(1+3\alpha)}{(1+\theta/\tau)^2}\\&-\frac{\alpha^3}{2(1+\theta/\tau)^3}+\frac{\alpha(\alpha+3\alpha)^2}{2(1+2\theta/\tau)}-\frac{\alpha^2(1+6\alpha)}{4(1+2\theta/\tau)^2}+\frac{\alpha^3}{2(1+2\theta/\tau)^3}\\&-\frac{\alpha^3}{2(1+3\theta/\tau)}+\frac{\alpha^3}{2(1+3\theta/\tau)^2}-\frac{\alpha^3}{6(1+3\theta/\tau)^3}\Big)\end{aligned}\tag{3.2}$$

The third segment 3 corresponds to the time interval $10\theta<\tau<\infty$; here

$$\Delta p=-\frac{Q\mu}{4\pi kh}\,\mathrm{Ei}\left(-\frac{1}{4\tau}\right)\approx\frac{Q\mu}{4\pi kh}(\ln 2.25+\ln\tau)\tag{3.3}$$

The approximate form of the theoretical pressure recovery curve is shown in the figure. The asymptote CD corresponds to lower compressibility and greater piezoconductivity of the formation (formation roof and floor still stationary), the asymptote AB corresponds to maximal compressibility of the

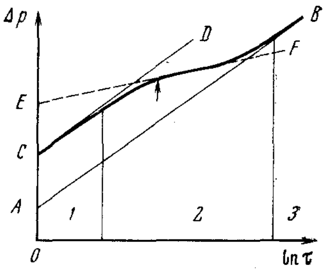

Fig. 1

formation (formation roof and floor compress the skeleton of the porous medium); both asymptotes are described by the same traditional formulas for the elastic filtration regime (3.1) and (3.3) with the same conductivity kh/μ but with different effective piezoconductivity parameters ($\varkappa/(1-\alpha)$ and $\varkappa$).

Let us make some numerical estimates. Let d = 20 mm, r_W = 10 cm, $\varkappa = 10^4$ cm^2/sec. Then $\theta = 10^4$ and the duration of interval 1 will be $\tau_1 = 0.1\ \theta = 10^3$ or t = 10 sec. The segment 3 begins at $\tau_2 = 10\ \theta = 10^5$ or at t = 1000 sec. Thus the duration of the transitional segment 2 will be on the order of 17 min.

We see from this example that segment 1 can in general be omitted entirely in practical measurements on wells, and segement 2 of the curve can be the primary object of observation. In so doing the first half of this segment (up to the arrow in the figure) can be taken as the defective part of the curve (for example, because of noninstantaneous shutdown of the well), and the second part (after the arrow) can be taken as the asymptote AB. Then drawing the false asymptote EF (dashed) from the experimental points leads to overestimation of the piezoconductivity coefficient $\varkappa$ and underestimation of the conductivity kh/ρ in comparison with their actual values.

In the case of low-permeability formations (small $\varkappa$) segment 1 may be recorded on the pressure recovery curves. Then the transition to segment 2 (up to the arrow) can be mistakenly taken as the asymptote corresponding to traditional elastic filtration regime theory.

In conclusion we emphasize that for very long pressure recovery times the assumption of stationarity of the initial pressure distribution is no longer satisfied. The required correction, as in known [5, 6], lies in plotting the curve of Δp versus $\ln[\tau/(\tau + T)]$, where T is the time of test well operation up to the moment of shutoff.

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